Extrinsic Calibration of Multiple Inertial Sensors From Arbitrary Trajectories

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INTRODUCTION

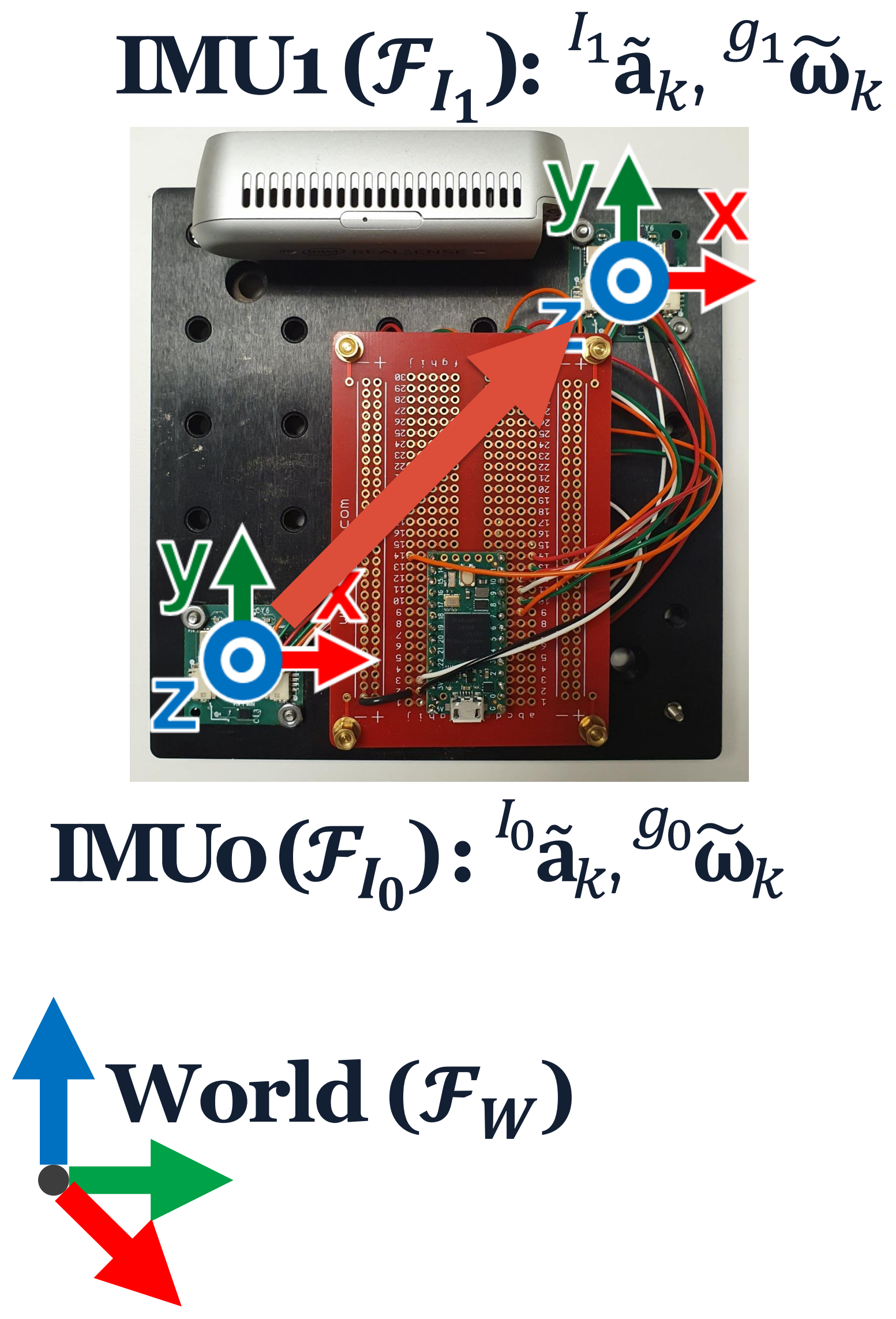
- The simultaneous use of multiple IMUs—as compared with a single IMU of the same total size, weight, power, and cost—may have three benefits:
  - Higher measurement accuracy
  - Increased bandwidth
  - Better fault tolerance
- It is necessary to perform *extrinsic calibration*—to estimate the relative position and orientation of each IMU—in order to realize these benefits
- We propose a method of extrinsic calibration for multiple IMUs that does not require instruments (e.g., rate tables) or aiding sensors (e.g., a camera) and that can be applied to data collected in-flight along arbitrary trajectories
- Our method is based on solving a nonlinear least-squares problem that penalizes inconsistency between measurements from pairs of IMUs

METHOD

- Each IMU's measurement is modeled as:
  - Accelerometer: $^{I}\tilde{\mathbf{a}} = \boxed{{}^{I}\mathbf{a}_{WI} - {}^{I}\mathbf{g}} + \mathbf{b}_a + \mathbf{n}_a$
  - Gyroscope: $^{g}\tilde{\boldsymbol{\omega}} = \boxed{\mathbf{C}({}^{g}_{I}\mathbf{q})\ {}^{I}\boldsymbol{\omega}_{WI}} + \mathbf{b}_g + \mathbf{n}_g$

specific force of frame I with respect to frame W expressed in frame I
time-varying bias described by random walk; $\mathbf{b}_{a,\,k+1} - \mathbf{b}_{a,\,k} \sim \sigma_{\mathbf{b}_a}\sqrt{\Delta t}\cdot\mathcal{N}(\mathbf{0},\mathbf{1})$
white noise; $\mathbf{n}_a \sim \sigma_a/\sqrt{\Delta t}\cdot\mathcal{N}(\mathbf{0},\mathbf{1})$
angular rate of frame I with respect to frame W expressed in frame g $(= {}^{g}\boldsymbol{\omega}_{WI} = {}^{g}\boldsymbol{\omega}_{Wg})$
time-varying bias described by random walk; $\mathbf{b}_{g,\,k+1} - \mathbf{b}_{g,\,k} \sim \sigma_{\mathbf{b}_g}\sqrt{\Delta t}\cdot\mathcal{N}(\mathbf{0},\mathbf{1})$
white noise; $\mathbf{n}_g \sim \sigma_g/\sqrt{\Delta t}\cdot\mathcal{N}(\mathbf{0},\mathbf{1})$
unit quaternion ${}^{g}_{I}\mathbf{q} \in \mathbb{H}$ parametrizing orientation of frame I with respect to frame g (i.e., gyroscope misalignment)
operator converting unit quaternion ${}^{g}_{I}\mathbf{q} \in \mathbb{H}$ to corresponding rotation matrix ${}^{g}_{I}\mathbf{R} \in SO(3)$

* Δt: sampling interval; (σ_a, σ_g): square root of noise spectral density; $(\sigma_{\mathbf{b}_a}, \sigma_{\mathbf{b}_g})$: bias instability


IMU1 ($\mathcal{F}_{I_1}$): $^{I_1}\tilde{\mathbf{a}}_k,\ ^{g_1}\tilde{\boldsymbol{\omega}}_k$

IMU0 ($\mathcal{F}_{I_0}$): $^{I_0}\tilde{\mathbf{a}}_k,\ ^{g_0}\tilde{\boldsymbol{\omega}}_k$

World ($\mathcal{F}_W$)

- Given: $^{I_0}\tilde{\mathbf{a}}_k, {}^{g_0}\tilde{\boldsymbol{\omega}}_k, {}^{I_1}\tilde{\mathbf{a}}_k, {}^{g_1}\tilde{\boldsymbol{\omega}}_k$ for each time $k \in \{1, \dots, K\}$
- To find: $^{I_0}\mathbf{p}_{I_0 I_1} := \mathbf{p} \in \mathbb{R}^3, {}^{I_0}_{I_1}\mathbf{q} := \mathbf{q} \in \mathbb{H}, {}^{g_0}_{I_0}\mathbf{q} \in \mathbb{H}, {}^{g_1}_{I_1}\mathbf{q} \in \mathbb{H},$

$$\underbrace{\mathbf{b}_{a_0,\,k}, \mathbf{b}_{g_0,\,k}, \mathbf{b}_{a_1,\,k}, \mathbf{b}_{g_1,\,k}, {}^{I_0}\boldsymbol{\alpha}_{WI_0,\,k} := {}^{I_0}\boldsymbol{\alpha}_k}_{\text{slack variables; their estimation is not main focus}}$$

- $$\min \sum_k \begin{pmatrix} \|\mathbf{r}_a\|^2_{\boldsymbol{\Sigma}_a} + \|\mathbf{r}_g\|^2_{\boldsymbol{\Sigma}_g} \\ + \left\|\mathbf{r}_{\mathbf{b}_{a_0}}\right\|^2_{\boldsymbol{\Sigma}_{\mathbf{b}_a}} + \left\|\mathbf{r}_{\mathbf{b}_{a_1}}\right\|^2_{\boldsymbol{\Sigma}_{\mathbf{b}_a}} + \left\|\mathbf{r}_{\mathbf{b}_{g_0}}\right\|^2_{\boldsymbol{\Sigma}_{\mathbf{b}_g}} + \left\|\mathbf{r}_{\mathbf{b}_{g_1}}\right\|^2_{\boldsymbol{\Sigma}_{\mathbf{b}_g}} \end{pmatrix}$$

specific force reconstructed by IMU0 measurement
accelerometer measurement of IMU1
accelerometer bias of IMU1

- $\mathbf{r}_a = {}^{I_1}\hat{\mathbf{a}}_k - ({}^{I_1}\tilde{\mathbf{a}}_k - \mathbf{b}_{a_1,k})$

$^{I_1}\hat{\mathbf{a}}_k = \mathbf{C}({}^{g_1}_{I_1}\mathbf{q})^{-1}\left({}^{I_0}\hat{\mathbf{a}}_k + [{}^{I_0}\hat{\boldsymbol{\omega}}_k]^2_\times \mathbf{p} + [{}^{I_0}\boldsymbol{\alpha}_k]_\times \mathbf{p}\right)$, where $^{I_0}\hat{\mathbf{a}}_k = {}^{I_0}\tilde{\mathbf{a}}_k - \mathbf{b}_{a_0,k}$ and $^{I_0}\hat{\boldsymbol{\omega}}_k = \mathbf{C}({}^{g_0}_{I_0}\mathbf{q})^{-1}({}^{g_0}\tilde{\boldsymbol{\omega}}_k - \mathbf{b}_{g_0,k})$

angular rate reconstructed by IMU0 measurement
gyroscope measurement of IMU1
gyroscope bias of IMU1

- $\mathbf{r}_g = {}^{g_1}\hat{\boldsymbol{\omega}}_k - ({}^{g_1}\tilde{\boldsymbol{\omega}}_k - \mathbf{b}_{g_1,k})$

$^{g_1}\hat{\boldsymbol{\omega}}_k = \mathbf{C}({}^{g_1}_{I_1}\mathbf{q})\,\mathbf{C}(\mathbf{q})^{-1}\mathbf{C}({}^{g_0}_{I_0}\mathbf{q})^{-1}({}^{g_0}\tilde{\boldsymbol{\omega}}_k - \mathbf{b}_{g_0,k})$

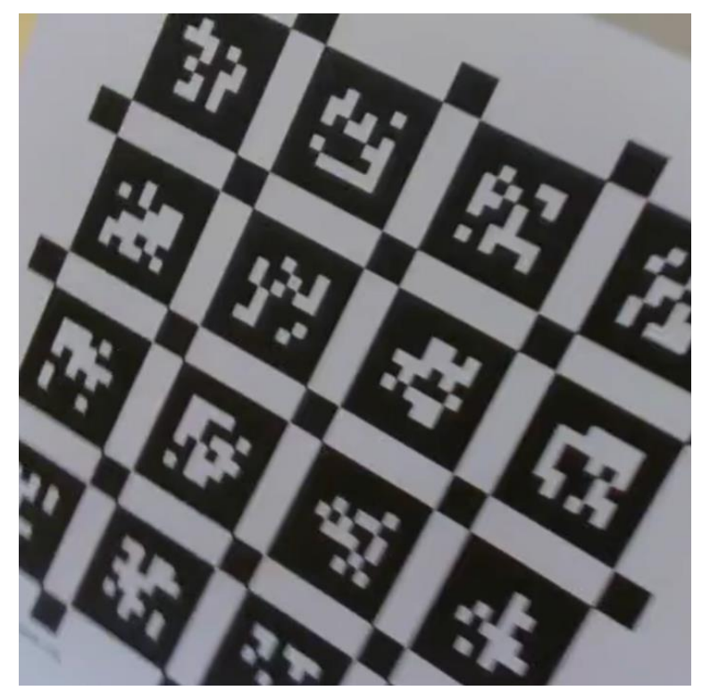
- $\mathbf{r}_{\mathbf{b}_{a_0}} = \mathbf{b}_{a_0,\,k+1} - \mathbf{b}_{a_0,\,k},\ \ \mathbf{r}_{\mathbf{b}_{a_1}} = \mathbf{b}_{a_1,\,k+1} - \mathbf{b}_{a_1,\,k}$
- $\mathbf{r}_{\mathbf{b}_{g_0}} = \mathbf{b}_{g_0,\,k+1} - \mathbf{b}_{g_0,\,k},\ \ \mathbf{r}_{\mathbf{b}_{g_1}} = \mathbf{b}_{g_1,\,k+1} - \mathbf{b}_{g_1,\,k}$
- $\boldsymbol{\Sigma}_a = \left\{2\sigma_a^2/\Delta t + (\sigma_g^2/\Delta t)^2\right\}\cdot \mathbf{I}_{3\times3},\ \boldsymbol{\Sigma}_g = 2\sigma_g^2/\Delta t \cdot \mathbf{I}_{3\times3},$

  $\boldsymbol{\Sigma}_{\mathbf{b}_a} = \sigma_{\mathbf{b}_a}^2\,\Delta t \cdot \mathbf{I}_{3\times3},\ \boldsymbol{\Sigma}_{\mathbf{b}_g} = \sigma_{\mathbf{b}_g}^2\,\Delta t \cdot \mathbf{I}_{3\times3}$

RESULTS

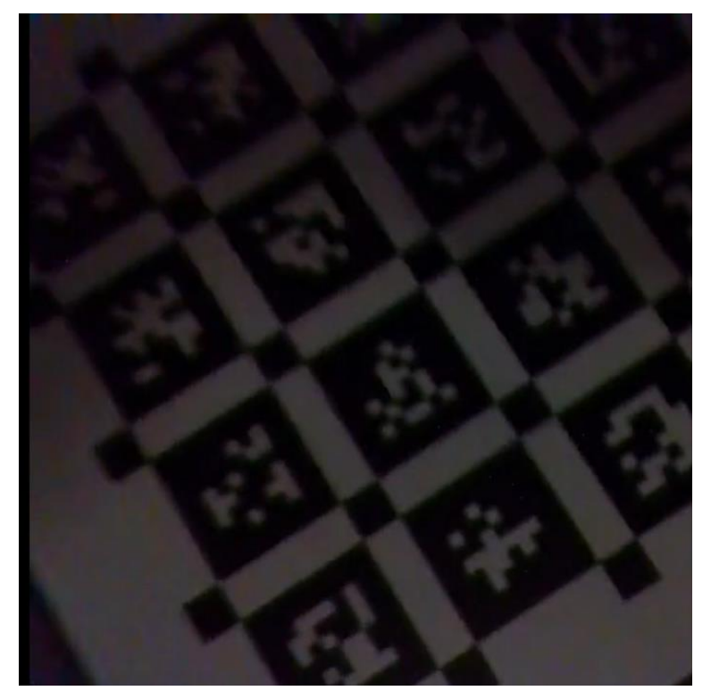
- We compare our method to Kalibr, a convenient benchmark for multi-IMU extrinsic calibration
- Kalibr needs a camera and fiducial marker, which accompany failure cases depending on visual conditions
- Both methods are tested over 65 trajectories (60 seconds for each) collected in three different conditions:

baseline (21 trajectories)



blurry (23 trajectories)

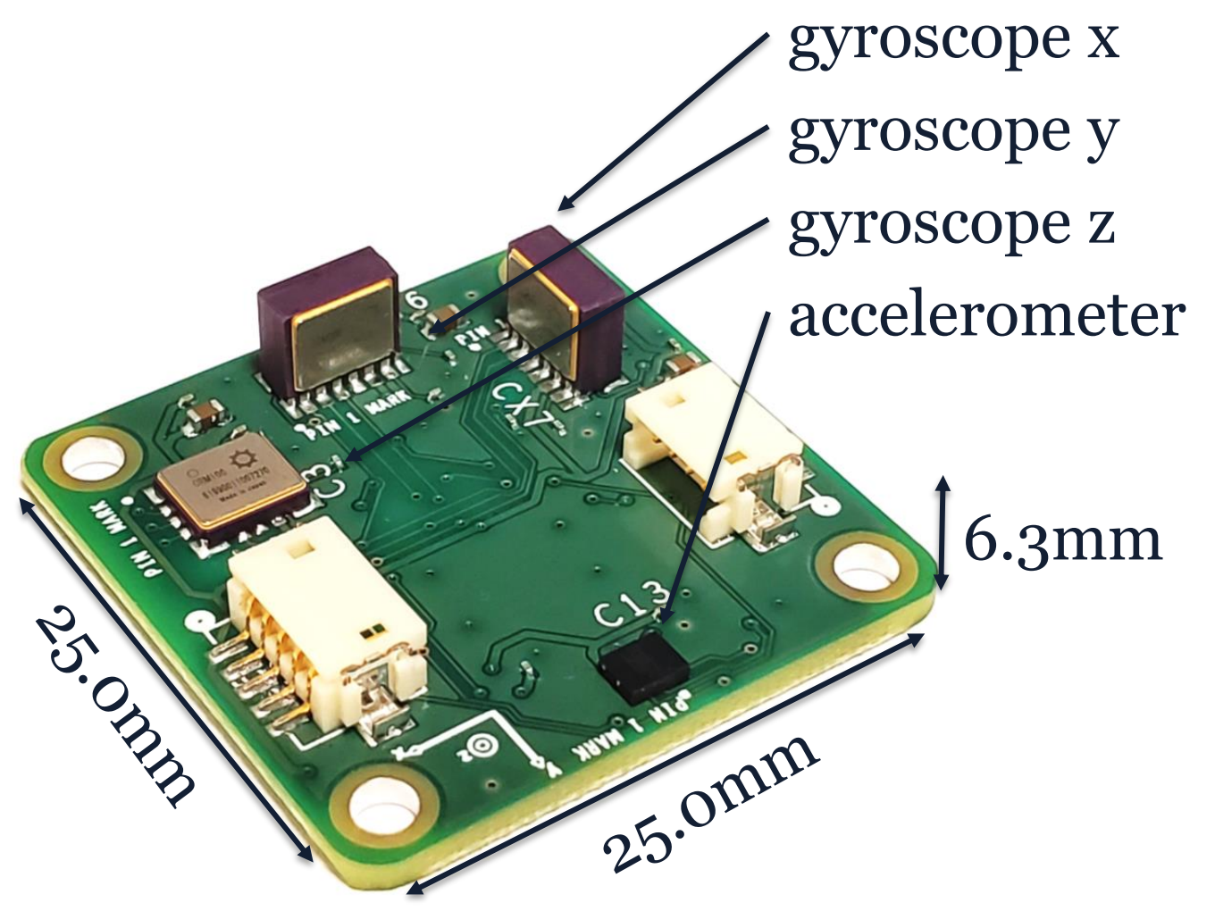
ill-lit (21 trajectories)



condition		**Kalibr**	**Our Method**
baseline	RMSE in $\mathbf{p}$ [mm]	4.39	1.64
	RMSE in $\mathbf{q}$ [deg]	1.35	2.91
	RMSE in ${}^{g}_{I}\mathbf{q}$ [deg]	2.06	2.50
	computation time [ms]	106.96 ± 16.34	4.61 ± 0.36
	success rate	19 / 21	21 / 21
blurry	RMSE in $\mathbf{p}$ [mm]	70.41	2.02
	RMSE in $\mathbf{q}$ [deg]	26.98	2.86
	RMSE in ${}^{g}_{I}\mathbf{q}$ [deg]	30.67	2.05
	computation time [ms]	98.99 ± 12.27	4.74 ± 0.58
	success rate	14 / 23	23 / 23
ill-lit	RMSE in $\mathbf{p}$ [mm]	107.42	1.37
	RMSE in $\mathbf{q}$ [deg]	1.81	4.19
	RMSE in ${}^{g}_{I}\mathbf{q}$ [deg]	1.03	3.51
	computation time [ms]	7.83 ± 0.28	5.16 ± 0.55
	success rate	2 / 21	21 / 21

$\mathbf{p}$: relative position, $\mathbf{q}$: relative orientation, ${}^{g}_{I}\mathbf{q}$: gyroscope misalignment

$\mathbf{p}_{\text{ref}}: [100, 100, 0] \pm [25.0, 25.0, 6.3]$ mm, $\mathbf{q}_{\text{ref}}, {}^{g}_{I}\mathbf{q}_{\text{ref}}: (\mathbf{e}, 0^\circ) \pm (\mathbf{e}, 9.5^\circ)$ for $\forall\, \mathbf{e} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$ (angle-axis)



IMU used for the experiments

- We acknowledge that the reference value we used—$\mathbf{p}_{\text{ref}}$ and $\mathbf{q}_{\text{ref}}$ derived from the optical breadboard on which sensors are mounted and zero gyroscope misalignment ${}^{g}_{I}\mathbf{q}_{\text{ref}}$—can be a source of error themselves
- Hence, we focus on establishing that both our method and Kalibr produce "comparable" errors with respect to these reference values, rather than on rigorously establishing that our method produces "lower" error
- Under the baseline condition, our method matches the performance of Kalibr
- Under the blurry and ill-lit conditions, Kalibr often fails, whereas our method is unaffected

CONCLUSIONS AND FUTURE WORK

- We proposed a method of extrinsic calibration for multiple IMUs that only uses measurements collected by the IMUs themselves along arbitrary trajectories
- We used all available measurements for extrinsic calibration—in future, it may be helpful to choose a subset of measurements in order to decrease computation time and increase robustness
- We assumed measurements were time-synchronized—in future, it may be possible to include time synchronization as part of the extrinsic calibration process

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All data and code are freely available online: https://github.com/jongwonlee/mix-cal